DO NOW

Find the second derivative of: $y^2 = x^2 + 2x$

$2yy' = 2x+2$

$y' = \frac{2x+2}{2y}$

$y' = \frac{2(x+1)}{2y}$

$y' = \frac{(x+1)}{y}$

$y' = (x+1)y^{-1}$

$y'' = (x+1)\cdot -1y^{-2}y' + y^{-1}(1)$

$y'' = -\frac{(x+1)}{y^2}y' + \frac{1}{y}$

$y'' = -\frac{(x+1)}{y^2}\cdot\frac{x+1}{y} + \frac{1}{y}$

$y'' = -\frac{(x+1)^2}{y^3} + \frac{1}{y}$

$y'' = \frac{-(x+1)^2 + y^2}{y^3}$

$y'' = \frac{-(x^2+2x+1) + x^2+2x}{y^3}$

$\boxed{y'' = \frac{-1}{y^3}}$

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3.8 Newton's Method

**Used for: approximate the real zeros (roots) of a function

- Is like: Solving by factoring, completing the square, or quadratic equation.

★This can be used when other methods cannot!!!

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Recall how to write the equation of a tangent line using the derivative:

point: $(x_0, f(x_0))$ $\quad m = f'(x_0)$

$y - y_1 = m(x - x_1)$

$y - f(x_0) = f'(x_0)(x - x_0)$

$y = f(x_0) + f'(x_0)(x - x_0) \leftarrow$ equation of the tangent line

Find x_1 if $(x_1, 0)$ is a point on the line.

$0 = f(x_0) + f'(x_0)(x_1 - x_0)$

$-f(x_0) = f'(x_0)(x_1 - x_0)$

$\frac{-f(x_0)}{f'(x_0)} = x_1 - x_0$

$\boxed{x_0 - \frac{f(x_0)}{f'(x_0)} = x_1} \leftarrow$ formula

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Let $f(c) = 0$, where f is differentiable on an open interval containing c. Then, to approximate c, use the following steps.

1. Make an initial guess x_1 that is close to c. (A graph is helpful.)
2. Determine a new approximation using: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
3. If $|x_n - x_{n+1}|$ is within the desired accuracy, let x_{n+1} serve as the final approximation. Otherwise, repeat step 2.

★★ Each successive application is called an: iteration.

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Example: Complete two iterations of Newton's Method for the function using the given initial guess.

$f(x) = 2x^2 - 3, \quad x_1 = 1$

$f'(x) = 4x$

I	x_n	$f(x_n)$	$f'(x_n)$	$\frac{f(x_n)}{f'(x_n)}$	$x_n - \frac{f(x_n)}{f'(x_n)}$
1	1	-1	4	$-\frac{1}{4} = -.25$	1.25
2	1.25	.125	5	.025	1.225

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2. Use Newton's Method to approximate the zeros of: $f(x) = e^x + x$. Continue iterations until two successive approximations differ by less than 0.0001.

$f'(x) = e^x + 1$

I	x_n	$f(x_n)$	$f'(x_n)$	$\frac{f(x_n)}{f'(x_n)}$	$x_n - \frac{f(x_n)}{f'(x_n)}$
1	-.5	.10653	1.60653	.06631	-.56631
2	-.56631	.00131	1.56761	.00084	-.56715
3	-.56715	-.00001	1.56714	.00000	-.56715

$\boxed{\text{Approximately } -.56715}$

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HOMEWORK

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